Compositional Systems: Overview and Applications

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Abstract: *In this paper the theory of compositional systems is described in detail, taking as a starting point the theoretical framework inherent to systems science. The origins of this science and the definitions of its fundamental concepts are provided in the first part of the article, illustrated with musical examples. The central part of the article contains the definition of the concept of compositional system, its typology, and a series of tools that are useful for implementations. Finally, the design of three types of systems (open, semi-open and feedback) are carried out in order to produce small illustrative musical fragments.*

Keywords*: Compositional Systems. Systems Science. Systemic Modeling. Probability.*

I. Introduction

Systems are ubiquitous in various human activities. However, despite the common-sense familiarity with systems in our everyday life, its concept is far from trivial and the attempts to define it has given rise to a large amount of studies, trends, and quarrels[1]. As pointed out by Robert Rosen [35], "the word system is never used by itself; it is generally accompanied by an adjective or other modifier: physical system; social system", etc. In this paper I am concerned with a special type of system: the compositional system. I start with a brief survey on systems theory, which includes historical aspects and definitions. This will lay the theoretical framework so that the concept of compositional system can be introduced and musical implementations can be performed.

II. A brief survey on systems theory

In this section, I will introduce some historical marks and basic concepts associated with the notion of system, in the general sense, and lay the foundations for the next section, which deals with a particular type of system: the compositional system. For the sake of clarity, I will demonstrate some systems concepts with musical examples, although the literature on systems focus especially on highly complex structures, such as living organisms, society, or even reality. This section will cover the motivations for the development of systems science as well as some of its historical marks (**??**) and definitions of the concept of system (i).

The roots of systems science are mathematics, computer technology, and a group of ideas known as systems thinking [16, p.19]. It emerged from the necessity of dealing with organized

[1] Lars Skyttner [38] has organized a survey on several trends in the field of systems science that includes Klir, Boulding, Laszlo, and many others.

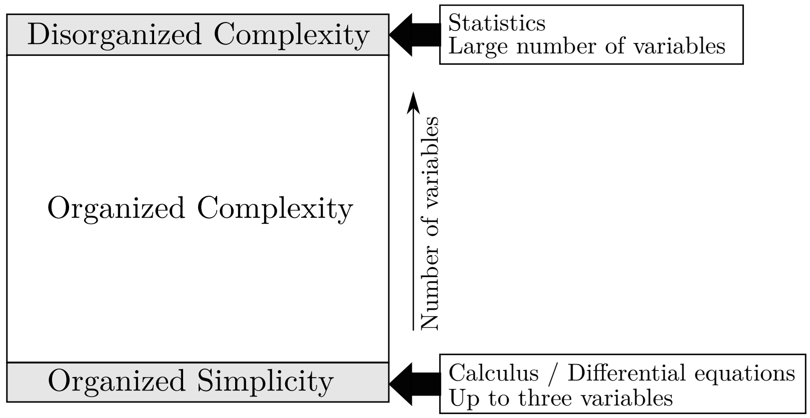


Figure 1: *Organized complexity between its extremes: organized simplicity and disorganized complexity.*

complexity, a category in which is located the vast majority of human problems, including music analysis and composition. In the extremities of this category are, on one side, organized simplicity, which consists of deterministic problems, involving up to four variables, that can be handled, for example, by calculus and differential equations, and, on the other side, disorganized complexity, which involves the use of probability and statistics to deal with an astronomical number of variables (Figure 1). As pointed out by Weaver, in the first half of the 20th century,

> One is tempted to oversimplify, and say that scientific methodology went from one extreme to the other—from two variables to an astronomical number—and left untouched a great middle region. The importance of this middle region, moreover, does not depend primarily on the fact that the number of variables involved is moderate—large compared to two, but small compared to the number of atoms in a pinch of salt. The problems in this middle region, in fact, will often involve a considerable number of variables. The really important characteristic of the problems of this middle region, which science has as yet little explored or conquered, lies in the fact that these problems, as contrasted with the disorganized situations with which statistics can cope, show the essential feature of organization. In fact, one can refer to this group of problems as those of organized complexity [42, p.539].

The limitation of dealing with a great number of variables is a cognitive feature inherently human. According to Halford et al [13, p.70] "a structure defined on four variables is at the limit of human processing capacity".[2] Moreover, structures with five variables are already at the chance level. A computer can make it easier to investigate complex systems involving a large number of variables. Therefore, the computer became a fundamental tool for investigating systems in the realm of organized complexity and its evolution has had a clear impact on systems science.

Besides mathematics and computer technology, a body of ideas connected with systems thinking were crucial to the emergence of systems science. Those ideas include holism, isomorphism, general systems, and cybernetics. Holism, an antithesis of reductionism[3], already known to Greek and Chinese philosophy, reappeared at the beginning of the 20th century in a branch of psychology known as Gestalt theory. It became present also in the field of biology, around the same time, in the *organismic biology* proposed by Paul Weiss and Ludwig von Bertalanffy [3]. Phillips [29, p.6-7]

[2] In music analysis and composition variables may be associated with attributes or parameters of a musical sound: pitch (pitch class and register), rhythm (attack point and duration), dynamics, articulation, and timbre. I have proposed the expansion of parametric repertory by introducing the concept of abstract parameter that includes inversional axis, rhythmic partition, degree of harmonic endogeny [32], melodic contour, and so on [33]. Those abstract parameters (except contour) are not easily detected by perception in the superficial level.

[3] Linked with the analytical method, i.e., a piecemeal approach in which an object is divided into its simple constituent elements.

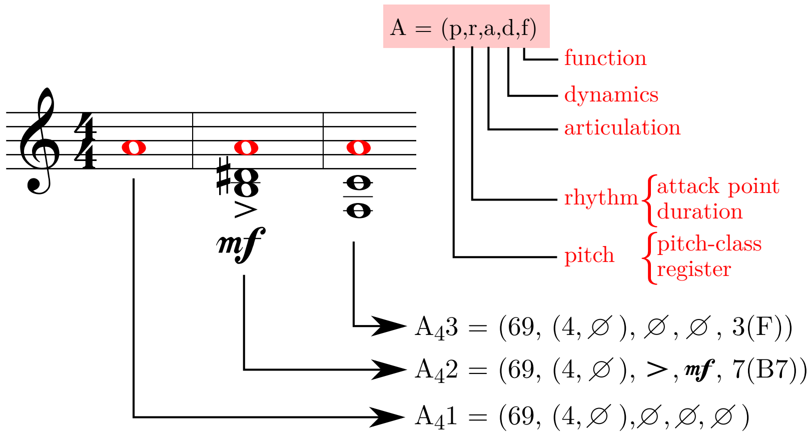


Figure 2: *Three situations in which pitch A4 appears: as an isolated pitch, as part of a B7 chord, and as part of an F7 chord.*

connects holism directly with Hegel's principle of internal relations and, thus, with 19th century organicism. He enumerates its five characteristics[4] :

1. The analytic approach as typified by the physicochemical sciences proves inadequate when applied to certain cases—for example, to a biological organism, to society, or even to reality as a whole.
2. The whole is more than the sum of its parts.
3. The whole determines the nature of its parts.
4. The parts cannot be understood if considered in isolation from the whole.
5. The parts are dynamically interrelated or interdependent.

The principle of internal relations appears as well in the writing of neo-idealists like Francis Bradley (1846-1924), who enunciates three key points: 1) Relations between entities are possible only inside a whole; 2) Those related entities are altered by the relationships; and 3) Those entities qualify the whole, which, in its turn, qualify them [29, p.8].

The second point is "the heart of the theory of internal relations" [29, p.8] and will be exemplified here by a musical note in three out-of-time contexts[5]. In the first situation, pitch A_4 is isolated from any system. In order to categorize it, let us assign to it the following parameters: pitch (pitch class and register)[6], rhythm (duration and attack point), articulation, dynamics, and function, (p,r,a,d,f). Therefore, this isolated A_4, which will be labeled A_41, can be represented by the expression $A_41 = (69, (4, \varnothing), \varnothing, \varnothing, \varnothing)$, meaning that it has information on pitch (MIDI number 69) and duration (4 quarter-notes), but no information on attack point, articulation and dynamics. Also, its isolated state deprives it of a chordal function. This is shown in Figure 2. In the second situation, the same A_4 plays the role (or has the function) of the seventh of a B7 chord and will be assigned articulation and dynamic values. It is ontologically the same A_4 but it is a different entity in the context of the system[7] formed by the other pitches encapsulated to form a more complex entity: the B7 chord. Its representation, therefore, will be $A_42 = (69, (4, \varnothing), >, mf, \varnothing)$. Figure 2

[4]Throughout his book, Denis Phillips, an adversary of inflexible holism, examines the validity of each one of those characteristics. He classifies Holisms in three types (p.36): I, II, and III. Holism I is the one firmly attached to the five aforementioned characteristics of organicism. Holism II and Holism III gradually accepts some kind of compromise with analytical methods.

[5]Contexts in which the attack (or time) points are not defined and therefore temporal order is disregarded.

[6]Or MIDI number (C_4 = 60)

[7]As it will be seen later in this paper, a system consists of interrelated objects.

shows yet another chord to which the A_4 belongs and functions as its third: F7. For Bradley, A_4 is a part of the B7 chord (which is a system), but when removed from it becomes simply an artifact, something like a dead and functionless entity.

Additionally, Phillips describes connections between systems theory and the philosophy of John Dewey (1859-1952)[8] , whose influence on musicologist Leonard Meyer (1918-2007) is already acknowledged in the literature [34][9]. Phillips also identifies association between systems theory and structuralism, especially in the works of Levy-Strauss (1908-2009)[10] and Jean Piaget (1896-1980)[11]. Structuralism has a methodological impact in the field of music analysis by introducing the synchronic perspective (as a complement to the diachronic one). As Piaget said, “structuralism is chiefly a departure from the diachronic study of isolated linguistic phenomena which prevailed in the nineteenth century and a turn to the investigation of synchronously functioning unified language systems” [30, p.4]. Lévy-Strauss adds:

> Hence the hypothesis: What is patterns showing affinity, instead of being considered in succession, were to be treated as one complex pattern and read as a whole? by getting at what we call harmony, they would then see that an orchestra score, to be meaningful, must be read diachronically along one axis—that is, page after page, and from left to right—and synchronically along the other axis, all the notes written vertically making up one grs constituent unit, that is, one bundle of relations. [20, p.212]

Another issue related to systems thinking is isomorphism (or analogies). In its etymology, isomorphism means simply a similarity of form. It is a concept that permeates several fields of knowledge such as biology, chemistry, sociology, and mathematics, for which it has a particular meaning: bijective correspondence and structure-preserving mappings[12]. In the field of music theory and analysis, for example, isomorphism is one of the formal supports for pitch class set theory, since pitch class space is isomorphic to the Abelian group $(Z, +)$ and, therefore, may inherit the algebraic structure related to groups.

Besides these internal aspects of isomorphism, systems science considers also a larger perspective on isomorphism, that is the connections among different areas. Klir [16, p.32] brings the example of generalized circuit, "a framework within which well-developed methods for analyzing electric circuits were transferred through established isomorphies to less advanced areas of mechanical, acoustic, magnetic, and thermal systems."

Consequently, isomorphism had as a natural result the increasing of interdisciplinarity, which, by its turn, led to the development of the concept of *general systems*, by Ludwig von Bertalanffy (1968), who was aligned with Kenneth Boulding, Ralph Gerard, and Anatol Rapoport. Klir [16, p.16] defines a general system as “a standard and interpretation-free[13] system chosen to represent a particular equivalence class of isomorphic systems”. The theory of general systems is extensively discussed in Bertalanffy’s book *General Systems Theory* [4].

Besides holism, isomorphism, and general systems, cybernetics was also a key factor for the development of systems science. According to Klir [16, p.37], "cybernetics is a subarea of general systems research that focuses on the study of information processes in systems, particularly

[8]Dewey and Bentley [8, p.509] mention that the world is historically presented to humans in three levels: 1) Self-action: isolated things; Inter-action: things in causal relationships; 3) things and relations forming an unbreakable whole. This last level is clearly related to the idea of system, as understood in a holistic fashion.

[9]Many references to the term system and style-system can be found in Meyer’s Emotion and Meaning in Music (1957) [26].

[10]As pointed out by Wilcken [44, p.140], “as its core, structural linguistics worked with a simple, yet revolutionary idea: the notion that language consisted of a formal system of interrelated elements, and that meaning resided not in the elements themselves, but in their relationships to one another.”

[11]For Piaget [30, p.5],“a structure is a system of transformations”.

[12]An isomorphic relation is equivalent, i.e., it is reflexive, symmetric, and transitive.

[13]Represented by integers or real numbers.

communication and control". It was created by Norbert Wiener, who defines it as "control and communication in the animal and in the machine"[14]. Cybernetics was highly benefited by the theory of information, developed by Claude Shannon and Warren Weaver. As pointed out by Eco [9], communication is an essential factor for cultural phenomena as well as for many scientific fields: Psychology, Genetics, Neurophysiology, etc.

i. Definitions of system

Probably, the earliest (explicit) definition of system comes from the French philosopher Étienne Bonnot de Condillac (1715-1780): "a system is nothing other than the arrangement of the different parts of an art or science in an order in which they all support each other, and where the latter are explained by the former. Those who give reason to others, are called principles; and the system is all the more perfect, as the principles are fewer: it is even to be hoped that they will be reduced to one"[15]. In his definition it is possible to clearly identify two aspects: parts and principles.

For Bertalanffy [4, p.55-56] "a system can be defined as a complex of interacting elements. Interaction means that elements, p, stand in relations, R, so that the behavior of an element p in R is different from its behavior in another relation, R'. If the behaviors in R and R' are not different, there is no interaction, and the elements behave independently with respect to the relations R and R'". Similarly to Condillac's definition, two aspects are also identified here: elements and relations. Bertalanffy went further and specified how should be the interaction of elements within a system. In order to illustrate his definition, consider the set of pitch classes $J = \{B, C\flat, D\sharp, E\flat, F\sharp, G\flat, A\}$ in which two quaternary relations [16] with one element each are defined: $R = \{(B, D\sharp, F\sharp, A)\}$ and $R' = \{(C\flat, E\flat, G\flat, A)\}$.[17] The behavior of pitch class A (the element p in Bertalanffy's definition) in R is different from its behavior in R', since R progresses harmonically to an E chord (set K) and R' progresses harmonically to a $B\flat$ chord (set K'). This is shown in Figure 3.

George Klir [16, p.4-9], inspired, according to himself, by a standard dictionary definition, proposes that a system is "a set or arrangement of **things** so **related** or connected as to form a unity or organic **whole**". I have highlighted in this definition three keywords: **things**, **related**, and **whole**. The latter corresponds to the system itself: it is the whole that emerges from the interaction of things and relations, which, in their turn, constitute the system's components. Klir formalizes this definition in Equation 1, in which S stands for system, T for things, and R for relation.[18] It is noteworthy to verify that for Klir the relational component (R) seems to be the essence of a system, since he associates it with the very property of systemhood. In other words, a set of unrelated things becomes a system when (and only when) these things are connected through some kind of relation. Figure 4A shows a collection of things (T1, T2, and T3). This collection of things is understood as a system when one finds relationships among them. In Figure 4B, the relations are

[14] This definition is on the very title of his 1948 book: Cybernetics or control and communication in the animal and in the machine [43]. In his book he covers topics such as the concept of time in Newtonian and Bergsonian terms, statistical mechanics, Gestalt theory, and information, language and society.

[15] In the original one can read: «un système n'est autre chose que la disposition des différentes parties d'un art ou d'une science dans un ordre où elles se soutiennent toutes mutuellement, et où les dernières s'expliquent par les premières. Celles qui rendent raison des autres, s'appellent principes ; et le système est d'autant plus parfait, que les principes sont en plus petit nombre : il est même à souhaiter qu'on les réduise à un seul » [7, p.1].

[16] A relation may be presented by enumerating its elements in the form of sets, through a matrix, a graph, or even, when the case applies, by its analytical expression.

[17] In the equal temperament tuning system applied to western concert instruments, like the piano, these two chords (represented by relations R and R') sound the same.

[18] T can be any arbitrary set, including the power set, or even other systems. A n-ary relation R on sets $A_1, A_2, ..., A_n$ is formally defined as a subset of a Cartesian product $A_1 \times A_2 \times, ..., \times A_n$.

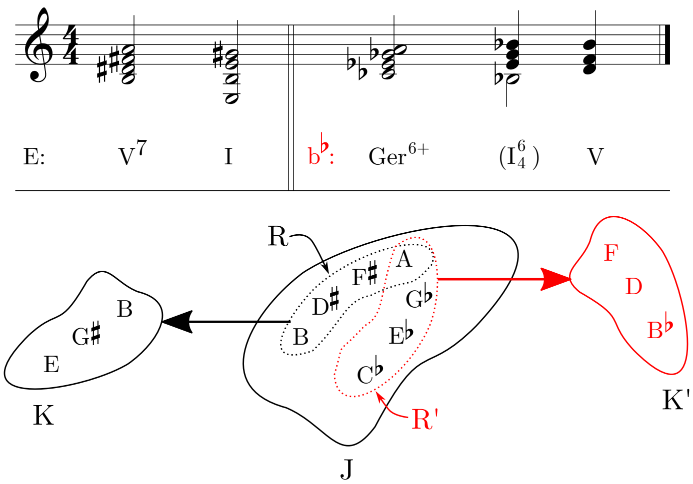


Figure 3: *pitch class A as part of a B7 chord (which resolves to an E chord) and as part of a German sixth chord in b♭ (which resolves to B♭). These chords correspond respectively to relation R and R′*

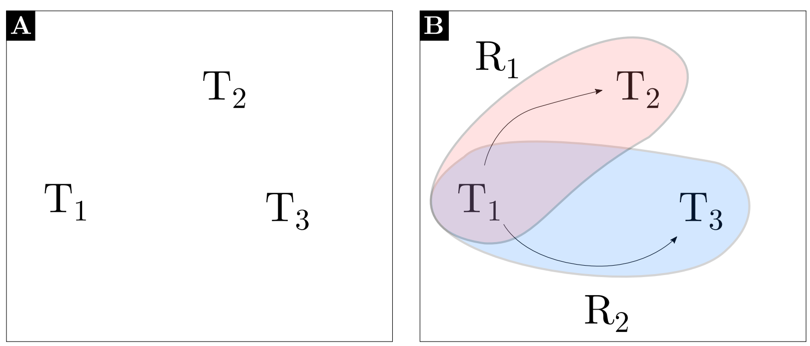


Figure 4: *In A, there is a set of things $\{T1, T2, T3\}$ and, in B, a system with three things $\{T1, T2, T3\}$ and two relations, $R1 = \{(T1, T2)\}$ and $R2 = \{(T1, T3)\}$.*

$R1 = \{(T1, T2)\}$ and $R2 = \{(T1, T3)\}$.

$$S = (T, R) \tag{1}$$

Thus, a collection of pitches in a musical score is not a system *per se*. Only when one identifies some kind of relationship among those pitches a system is cognitively established.[19] If fact, this identification is "the most fundamental act of system theory, the very act of defining the system presently of interest, of distinguishing it from its environment" [12, p.32]. Such identification is an individual task and depends upon the analytical repertory and also the particular choices of the observer. Therefore, different observers may define different systems from the same set of things.

The musical fragment shown in Figure 5 can be understood as different systems. A first analysis can understand the fragment as a melodic line in which the pitch content consists of a

[19]This is a key point in systems science: the musical score exists in the real world independently of our observation but our knowledge about it is only established through the epistemological attitude of making distinctions. This is, as mentioned by Klir [16, p.12], a constructivist perspective and could be traced as early as mid-17th century, in the works of Giambattista Vico (1668-1744). In the 20th century, the works of Jean Piaget (1896-1980), Ernst von Glasersfeld (1917 -2010), Humberto Maturana (1928-), and Francisco Varela (1946-2001) are connected with this epistemological view of the world.

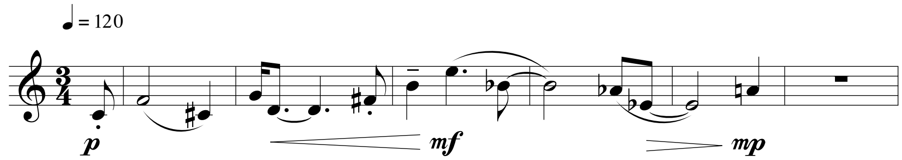


Figure 5: *A musical fragment with intervallic profile (+5,-4,+6,-5,+4,+5,+5,-6,-2,-5,+5).*

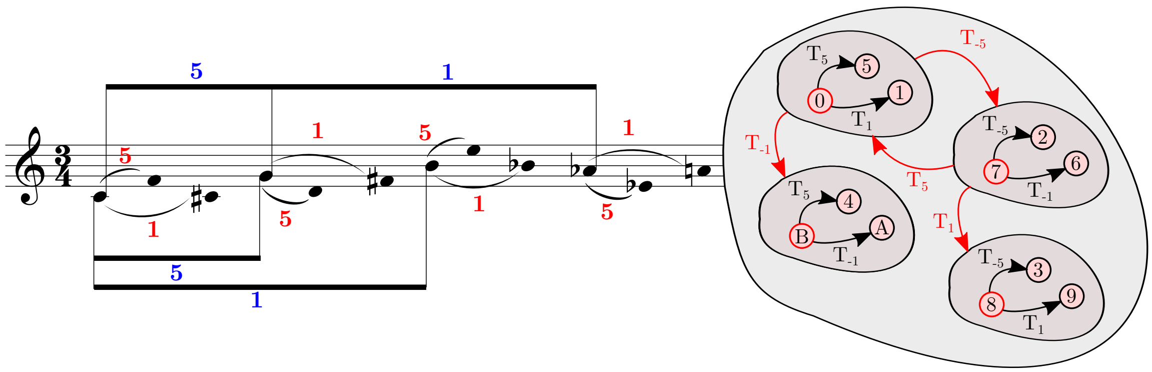


Figure 6: *The same musical fragment of Figure 5 understood now as a self-similar trichordal structure.*

twelve-tone row with intervallic profile $(+5, -4, +6, -5, +4, +5, +5, -6, -2, -5, +5)$.[20] Given the initial pitch, the entire collection can be reconstructed even though information about rhythm (duration and attack points), articulation, dynamics, and tempo is lost.

A second analysis could reveal inner relationships from which one perceives a self-similar structure. This structure is formed by trichords connected to each other through internal transpositional functions[21] that hold their elements together. Figure 6 shows that pitch class set $\{(0), 1, 5\}$[22], with 0 (inside parenthesis) arbitrarily defined as the main pitch class, and transpositional functions T_5 and T_1 form a system; set $\{(7), 2, 6\}$ and transpositional functions T_{-5} and T_{-1} form another system, and so on. A closer look at the four systems reveals that the interval class[23] from their main pitch class to the other two pitch classes are always 5 and 1. This means that all four systems consist of set class 015. Furthermore, the main pitch classes of the four systems relate to each other through the same transpositional functions used internally. Therefore, they can be grouped together to form a larger system similar to each one of its subsystems.

A third analysis could disregard absolute pitch or pitch class values and consider only the melodic contour of segments. In Figure 7, the melodic line was segmented into four parts and to each part was assigned a contour. It is easy to verify that they can all be related to the initial

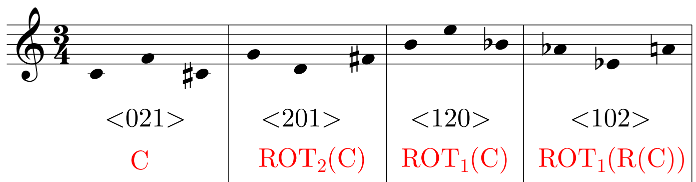
[20] The numbers inside parenthesis indicate the chromatic semitones between two pitches. Ascending and descending intervals are indicated with $+$ and $-$, respectively.

[21] According to Halmos [14, p.30], "if X and Y are sets, a function from (or on) X to (or into) Y is a relation f such that $dom f = X$ and such that for each x in X there is a unique element y in Y, with $(x, y) \in f$."

[22] In this paper, a pitch class set will be represented within braces, its normal form within parenthesis, and its prime form within brackets. A set class is represented unframed.

[23] The interval class (ic) is the smaller distance between two unordered pitch classes (a, b). Formally,

$$ic(a,b) = \begin{cases} 12 - |a-b| & \text{if } (a-b) > 6 \\ |a-b| & \text{if } b \leq 6. \end{cases} \quad (2)$$



C	<012>	R(C)	<210>
ROT$_1$(C)	<120>	ROT$_1$(R(C))	<102>
ROT$_2$(C)	<201>	ROT$_2$(R(C))	<021>

Figure 7: *The same musical fragment of Figure 3 understood now as chain of inter-related melodic contours.*

contour by rotation (*ROT*) and retrogradation (*R*).[24] Therefore, the second contour, <201> is the second rotation of the original contour, <021>, and so on.

Those three above systems were the result of examinations of a melodic line. From the observations I have proposed three different models.[25] However, a system can also be designed from scratch. In this paper, there is a differentiation between systemic designing and systemic modeling. The latter consists of proposing a model by capturing the architecture of a structure already in place with the purpose of understanding its rules of organization in such a way that it could be replicated or expanded; the former consists of defining a structure of objects and relations from the ground up.

Klir divides systems into five epistemological levels. The most basic level, the *source* system, consists of a set of variables (with no particular values) and potential states.[26] "When the source system is supplemented with data, i.e., with actual states of the basic variables within the defined support set" [17, p.13], the first level, the *data system*, is reached.[27]The data may come from modeling or designing. If information is given on the data generation (deterministic or stochastic), the system is on the *generative* level. So, at this level there are models capable of generating information. Higher levels include the *structure system*, which is a set of generative systems working cooperatively, and *metasystems*, which are systems that describe changes within structure systems (relations of relations).[28]

III. Compositional Systems

Approaches to musical composition through a systems science perspective, although rare, are already known in the scope of music research. Probably, the most comprehensive one is the extensive paper written in the 1990s by Romanian scholars Cosmin and Mario Georgescu [10], which brings strong and innovative associations between music and systems science. They identify several systemic features within a musical work: wholeness, hierarchical order, individualization, and centralization.[29] The last two are mostly useful to explain the appearance of stylistic common practices and the departure from them. They see the musical language as a result of stochastic

[24]Those operation will be formally defined later in this paper.

[25]"Model is a physical, mathematical, or otherwise logical representation of a system, entity, phenomenon or process"[2, p.3]. "Models in ordinary language have their place in systems theory. The system idea retains its value even where it cannot be formulated mathematically, or remains a 'guided idea' rather than being a mathematical construct" [4, p.24].

[26]This is also called a *dataless system* [16, p.49].

[27]It is interesting to find an example of data systems in terms of musical analysis in Klir [17, p.64-67].

[28]Metasystems are particularly important to monitor *morphogenetic systems*, a concept that will be defined below.

[29]A musical work, for them, "is a set of sound objects and processes, organized in a certain way so as to meet an objective–particular overall finalities of a communicational-aesthetic purport" [10, p.17].

procedures and fuzzy indeterminacy and highlight a strong contextual dependency of a musical work with its historical epoch, general trends and individual style. The authors consider that a musical system is, at least through the historical point of view, an open system[30], i.e., a system that interacts with the environment. In fact, they go even further and regard any musical composition as the result of tensions between a structural level and the environment, which produces bifurcation points, zones of uncertainty and fluctuations. Within this context, a key concept is presented: *morphogenetic music*. In contrast with structurally-stable music (the vast majority of Western concert music production), morphogenetic music has a structure that changes drastically over time. It is closely related to catastrophe theory, in which "abrupt changes of state are the result of smooth alteration of the control parameters" in dynamical systems [19, p.359][31]. In terms of Western concert music, the morphogenetic type is present in the transitional periods. If music leaves a steady-state, the bifurcation point leads either to negative entropy or to positive entropy. In the latter, a destructuring process moves towards a white noise state; in the former, a neostructuring process moves towards upper musical states. The operas *Agamenon* and *Oeumenides*, by Romanian composer Aurel Stroe, are respectively examples of both cases, according to the authors. It is noteworthy, however, that, in their article, the term "compositional system" appears only once and no definition is given[32].

A definition for the term *compositional system* was given (maybe for the first time) in Flávio Lima's dissertation [22, p.63], written under my supervision: a compositional system is "a set of guidelines, forming a coherent whole, which coordinates the use and interconnection of musical parameters, with the purpose of producing musical works". Later [31, p.69], I have proposed the inclusion of the word "materials" in the definition in order to considers also the materials as a whole, without breaking them into their various parameters. Therefore, the present definition of system is: a set of guidelines, forming a coherent whole, which coordinates the use and interconnection of musical parameters and materials, with the purpose of producing musical works. The idea of purpose (or function) is inspired by Meadows [24, p.11], for whom "a system must consist of three kinds of things: elements, interconnections, and a function or purpose".

A formal definition of compositional system S is given by the expression $S = (O, R)$, in which O corresponds to objects, i.e., materials or parameters (abstract or concrete) and R to relations (or functions, operations, and transformations). Differently from Klir's definition (Equation 1), relations here may be represented both in the form of subsets of a Cartesian product of the elements of O and by their analytical expressions.[33] Furthermore, the idea of purpose is embedded in the definition and clearly appears during the process of designing a system, as it will be seen next. Why was the word thing (stated in Equation 1 by Klir) translated as object in the initial moments of the foundation of the theory of compositional systems? During the bibliographic research phase for Flávio Lima's dissertation [22], I came across the book *Teoria dos Objetos*, by Abraham Moles [28]. In this book, Moles makes a distinction between the concepts of thing and object. The first is natural, the second is artificial, that is, produced by humans. Thus, as Moles says, a "stone will only become an object when promoted to paperweight, and when equipped with a label: price ..., quality ..., inserting it in the universe of social reference" [28, p.26]. In addition, Moles associates

[30]The concept of open system differs from the concept of open **compositional** system, as it will be seen later in this paper.

[31]Compositional experiments with catastrophe theory have been made by composers Ann Warde [41] and Fani Kosona [19].

[32]The authors frequently use the term "musical system", which appears 22 times.

[33]Relations are used for musical contexts in which an element of a certain domain is mapped onto two or more elements of the range. Functions are used for single parametric elements (a pitch class, a duration, etc.). Operations are functions applied to sets (a pitch class set, for example). Transformations are reserved for operations associated with Lewin's GMIT [21].

the term object with the concept of system when he states that "the whole set of elements or objects linked by functional relations can be considered as a system (...)" [28, p.28]. It is worth mentioning that this author collaborated with Pierre Schaeffer in the definition of the term *sound object*, which is a second use of the term object. Schaeffer divides this term into two categories: 1) musical object, which "is treated as the object of the language established between the composer and the listener (...). It is the spokesman for the musical language" [25, p.59]; 2) sound object, which is related to the sound itself (as much of the so-called musical sounds as of the noises). "The sound object emerges in the functions 'to perceive aurally / to hear (to pay attention)', while the musical object, inserted in a language, fits in the function 'to understand'"[25, p.65].[34]. Schaeffer relies heavily on Husserl's phenomenology to arrive at the essence of sound. A third strand for the term object can be found in computer science, more precisely in the programming paradigm known as OOP (object-oriented programming), which deals with the interaction between basic units called objects. Long after the foundation of the theory of compositional systems, there was a happy coincidence in the use of the term object (and not thing) in an article by Goguen [11] that deals with the formalization of musical systems (the second section of this work by Goguen is precisely entitled "Objects and Relations"). The term object in the theory of compositional systems is associated with very specific elements stated in the very fundamental definition of the theory. Objects are parametric structures (concrete, such as pitch, duration, register, harmonic entities, etc., or abstract, such as textural partitions, degree of harmonic endogeny, contours, inversional axes, etc.) or raw materials (fragments extracted from original works). The objects described in the theory of compositional systems encompass Schaeffer's sound and musical objects and are flexible to the point of encompassing, at least potentially, elements that do not directly involve sound, including elements of a spatial nature.

Compositional-system methodology considers the holistic phenomena (as described earlier in this paper) in a very loose manner—in a more flexible fashion that the third type of holism described in Phillips [29][35]. Therefore, analytical methods are largely employed and even regarded as essential procedures, especially for modeled systems. In terms of design, a compositional system may emerge from a series of formal declarations, diagrams, tables, and computational programs.

With respect to typology, I define three types of compositional systems: open, semi-open, and feedback [33]. Different combinations of these three types yield systems with higher complexity. It is noteworthy to emphasize that our classification is from a different nature when compared with systems science's typology, that is, it is not associated with the concepts of open and closed systems defined by Bertalanffy [4, p.121], for whom "a system [is] 'closed' if no material enters or leaves it; it is called 'open' if there is import and export of material"[36].

[34] Schaeffer defines four functions of listening: "écouter", "ouïr", "entendre", and "comprendre" [36]. I am using here the suggestions given by North and Dack in the 2017 English translation of the "Traité des objets musicaux"[37]

[35] See note 4

[36] The dicothomy open/closed systems in the context of social sciences is discussed in length by Luhmann [23]: "Physics has come to the understanding that the universe is a closed system that cannot accept any kind of input from an order that is not contained in itself and that, there, the law of entropy is inexorable. But if this is valid for the physical world, it is not the case for the biological or social order. Hence, the physical lock of the universe was denied as a phenomenon representative of other orders. So it was thought that these different systems would have to be fundamentally open, capable of developing neguentropia. This being open explained the effort of organisms (if you think of biology) to overcome, even partially, the entropic law of the universe. In the original one reads: "La física ha llegado a la comprensión de que el universo es un sistema cerrado, que no puede aceptar ningún tipo de input de un orden que no esté contenido en él mismo y que, allí, la ley de la entropía es inexorable. Pero si esto es válido para el mundo físico, no lo es, sin más, para el orden biológico ni el social. De aquí que la cerradura física del universo se negara como un fenómeno representativo de otros órdenes. Entonces se pensó que estos sistemas distintos tendrían que ser fundamentalmente abiertos, capaces de desarrollar neguentropía. Este ser abiertos explicaba el esfuerzo de los organismos (si se piensa en ia biología) por sobreponerse, aunque fuera parcialmente, a la ley entrópica del universo" [23, p.47].

Basically, in this methodology, open systems have input and output, semi-open systems have only output of data (although it may have some kind of control operators), and feedback systems have the data reinserted into its input. Before proceeding with designs for those three types of systems, it will be necessary to define some operations to be applied to musical parameters and materials.

i. Operations used in the design of compositional systems

In this subsection I will describe several operations that can be applied to parameters and materials in the realm of compositional systems. I will define fourteen operations, some of them original proposals that have already been experimented with compositional students and during my own compositional designs. The list is not exhaustive and their combination are encouraged in order to create compound operations. Some operations have effect only on the surface level (such as transposition, for example), while others (such as retrogradation) have a severe cognitive impact. Some of them are already incorporated in computer applications, such as the Lewin Calculator, developed by Barbosa, Santos, and Pitombeira [1].

1. Transposition (T_n) – rewriting of a segment at another pitch level. Intervallic relationships (and therefore contour) as well as rhythmic structure are preserved. It can be performed in terms of a diatonic set (in this case it is rotation through a modular space).
2. Inversion (I_n) – generates a mirrored outline (in a chromatic context) around the first pitch. The index n is a transpositional factor. It can also be performed diatonically.[37]
3. Prolation (P_t) – temporal expansion/contraction. It consists in rewriting a segment with longer or shorter durational values, according to factor t. Temporal expansion is traditionally known as augmentation and contraction as diminution.
4. Ambitus (A_i) – intervallic expansion/contraction. The line is rewritten with expanded or contracted intervals, according to factor i. Melodic contour and rhythm are preserved.
5. Retrogradation (R) – line (pitches and rhythmic values) rewritten backwards. One can also retrograde just one of the parameters. Although the original material can be visually identified, this transformation drastically changes the profile of the material from an auditory perspective, depending on the number of elements involved.
6. Rotation (ROT_k) – rotation of the elements of a pitch class set. The number of possible rotations depends on set cardinality.[38]
7. Multiplication – I use three types:
   (a) Boulez (Mb) – the intervallic profile of a set is applied to each element of another set.
   (b) Rahn (M_k) – a set (or an entire segment) is multiplied by a constant value (k).
   (c) Rahn expanded (Mr) – the elements of a pitch class set in normal form are concatenated to form an integer (base 12). For example: $J = \{10, 1, 2\} \rightarrow J = A12$; then, this set is multiplied by another set in the same format following the rules of regular arithmetic multiplication of two numbers.[39]

[37] This is different from the T_nI operation of the pitch class set theory, which first inverts around 0 and then applies the transpositional factor.

[38] Rotations and Reflections are the two types of Permutation of a dihedral group, such as a T_n/T_nI group. Reflections here are obtained by applying Rotations to Retrogradations.

[39] This is an original contribution of the present author to the multiplication of pitch class sets.

8. Addition/subtraction of elements (A/S) – insertion of elements (interpolation) or simplification of a line by removing elements.
9. Parametric fixation (Pf) – the pitch structure is maintained and the rhythmic structure is changed or contrariwise (without the obligation to maintain the same contour). Evidently, this concept can be applied to transformations that involve other parameters (density, contour, dynamics, articulation, ...).
10. Octave offset (Of) – consists of a free octave displacement applied to some pitches of a segment.
11. Filtering (Ft) – consists of filtering the notes through a different scale than the one used in the original construction (eliminating those that do not belong to the filter).
12. Conversion (C) – consists of filtering the notes through a scale different from that used in the original construction and converting the notes that do not belong to it (taking this new scale as a reference).
13. Permutation (P) – changes in the order of individual segments or elements.
14. Fragmentation (Fg) – free and repeated use of small portions of the original segment in its original or altered fashions.

I have designed a short example of an open compositional system (with input and output) using only the first six operators mentioned above: Transposition (T_k), Inversion (I_n), Prolation (P_t), Ambitus (A_i), Retrogradation (R), and Rotation (ROT_k). A diagram of this system is shown in Figure 8. The operators, except for R, need to be supplied with control factors (k, n, t, i, k). The first step consists of arbitrarily choosing a small musical fragment (original or taken from another work), in MIDI or *musicxml* format, and insert it into the system. Seven external keys controlled by the composer command the sequence of activation of each operator. They cannot be activated simultaneously. The original fragment as well as the output of each operator are appended to a temporal concatenator that sends the final stream to the system's output. The form of the piece is obtained by the gradual concatenation of the results of each operator. This system, implemented in Python, is a work in progress.[40] One possible musical result is shown at the bottom of Figure 8.

This output [41] may be used as the initial idea for a new composition, after some polishing, or even be reinserted into the system to generate a larger musical segment. I have chosen the first case, made small adjustments in the fragment, assigned it to a musical instrument (clarinet), and added a piano accompaniment. The adjustments are indicated in Figure 9: 1) Operator A/S (addition of elements) – two sixteenth notes (E and D) at the end of measure 2 to connect with the C in measure 3; 2) Rebar of measure 4 to follow the 3/4 metric; and 3) Operator T_{-4} (transposition a major third down) applied to ROT_1. For the piano, I have decided that its pitch classes come entirely from the clarinet's melody (except for measure 2, in which the embellishing note E was added to the original melodic line). Also, I have used the operation called Fragmentation (F) in order to assign the three first figures of measure 4 to the piano's left hand, but adapted to the harmonic constraints. The result is shown in Figure 9, in which the normal forms of the pitch class sets for each measure are indicated below the score.

The way harmony was generated for this excerpt[42] is called *Endogenous Harmony*.[43] It is possible to define other types of harmonic configurations in relation to a given melodic line.

[40] https://gitlab.com/musmat/open-compositional-system

[41] A midi rendition of this fragment is available at https://gitlab.com/musmat/open-compositional-system/-/blob/master/melodiaresultadosistema1.wav

[42] A midi rendition for this fragment is available at https://gitlab.com/musmat/open-compositional-system/-/blob/master/melodiaresultadosistema1harmonizada.wav

[43] This is a pedagogical tool created by the present author for his introductory compositional courses.

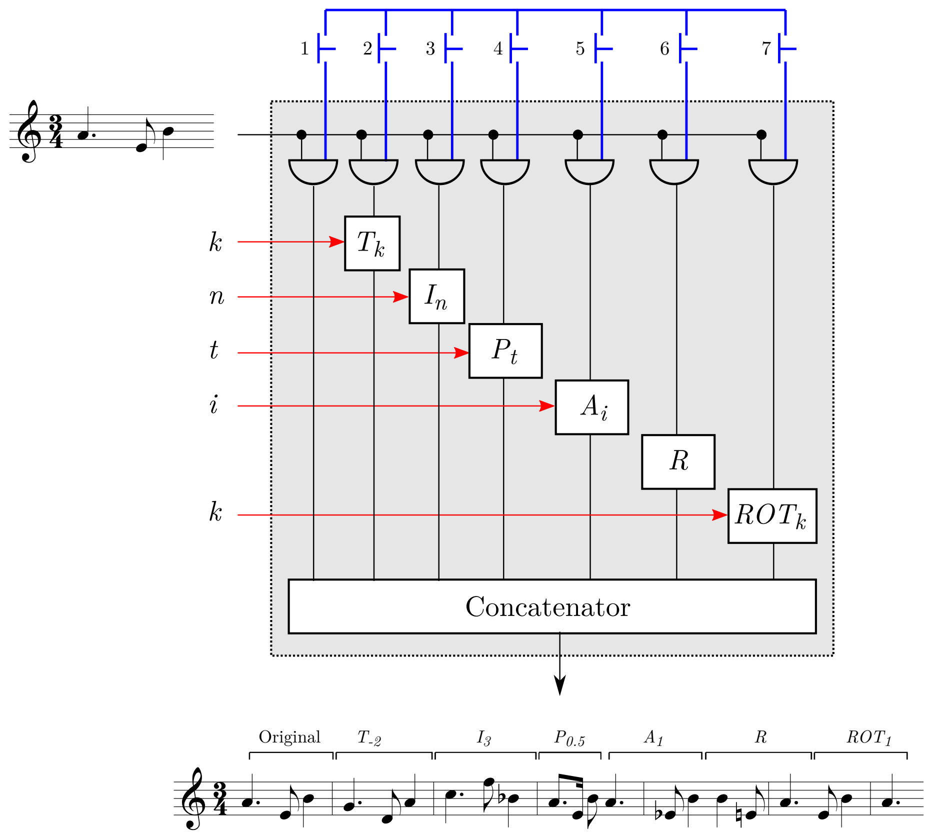


Figure 8: *Diagram of an open compositional system built with six operators and a temporal concatenator.*

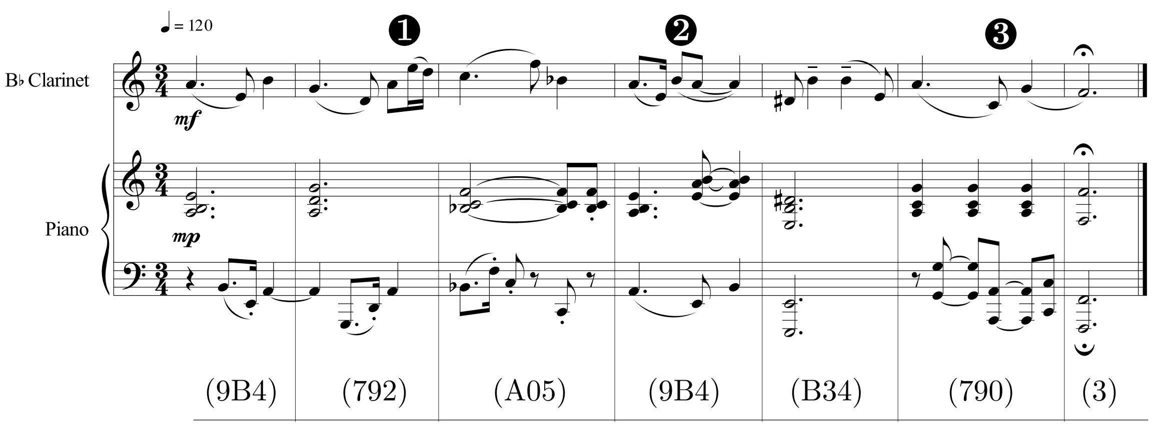


Figure 9: *Use of the fragment produced by the open compositional system in the beginning of a piece for piano and clarinet.*

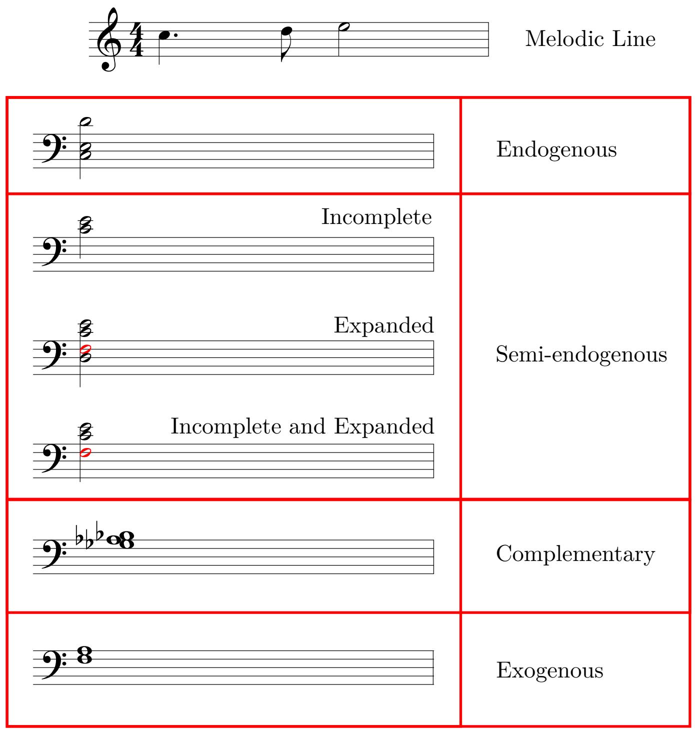


Figure 10: *Types of harmonic configurations derived from a given melodic line.*

Semi-endogenous harmonization takes place when the melody's pitch classes are partially applied to the harmony or when additional pitch classes are included. This type can be divided into three subtypes: a) Incomplete — the melodic pitch classes are partially used (in the example of Figure 10 only the pitch classes C and E are used in the harmony, leaving out the D, which is also part of the melodic line); b) Expanded — inclusion of additional notes (in the example Figure 10 all the pitch classes, but the added F, belong to the melodic line); c) Incomplete and Expanded – a combination of the previous subtypes (in Figure 10 the lower pitch class, F, does not belong to the melodic line, whereas the other two belong; but the D is missing). If the harmony is designed as a complement to the melody's pitch classes, with respect to some larger set, it is said that the harmony is *Complementary*. In the example of Figure 10 the harmony is the complement of the melody with respect to the whole tone scale $C, D, E, G\flat, A\flat, B\flat$. Finally, if the harmony does not have any connection with the melodic line it is called *Exogenous*.

As an example of a semi-open compositional system, I have designed a system in which the internal data is generated through probability: a binomial distribution for the pitch parameter and a uniform distribution for the rhythmic structure. In the uniform distribution all the outcomes have the same probability. The binomial distribution is a discrete distribution that counts the amount of "successes" or "failures" in binary experiments. If n is the number of trials of a probabilistic experiment, p is the probability of success of each outcome, and k is the number of desired success, the probability of k is given by Equation 3.

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{3}$$

For example, if one flips a coin eight times to obtain "heads", a binomial experiment is taking

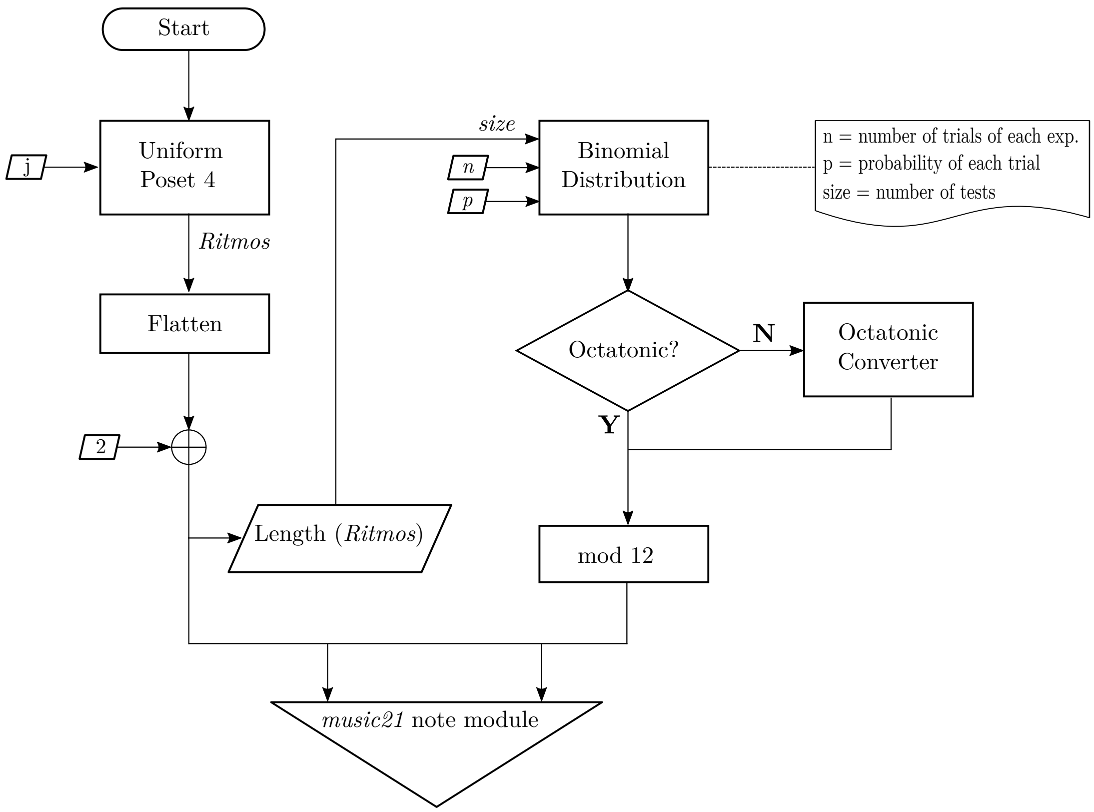


Figure 11: *Flowchart for a semi-open compositional system.*

place. Its parameters are 8 (number of trials) and 0.5 (probability that a head will occur in a single attempt). If one wishes to know the probability of having three successes the result will be given by Equation4:

$$P(X=3) = \binom{8}{3} 0.5^3 (1-0.5)^{8-3} \tag{4}$$

The flowchart for the semi-open compositional system is shown in Figure 11. It starts by uniformly randomizing a *poset* formed by the compositions[44] of integer 4, which are isomorphic to the eight possible integer subdivisions of a quarter note (Figure 12).[45] The composer chooses the number of randomizations (j). The result is transformed into a flat list, i.e., without the separation of rhythmic figures (which are already guaranteed through the ordered randomization). An arbitrary rhythmic figure of half note is appended at the end of the list (called *ritmos*) to promote a rhythmic cadence and the result is sent to the note module of the *music21* Python package.[46]

[44]Posets are partially ordered sets, i.e., sets in which the elements are ordered but not all of them required to be hierarchically comparable. Compositions are partitions in which the order is essential. The number of compositions (C) of an integer n is given by $C_n = 2^{n-1}$.

[45]The operation that connects the nodes from the bottom (1.1.1.1) towards the top composition (4) is the sum of consecutive parts. Therefore for a composition $C_i = p_1, p_2, \ldots, p_n$, with n parts, a composition C_j, with $n-1$ parts is$C_j = p_1, p_2, \ldots (p_k + (p_{k+1})), \ldots, p_n$. So, the composition 1.1.1.1 may progress to 2.1.1, which is the sum of its first and second parts $(1+1)$ or may progress to 1.2.1, which is the sum of its second and third parts, and so on.

[46]The *music21* package is a Python library to handle musical objects developed by Michael Cuthbert, at the MIT, and available at https://web.mit.edu/music21/

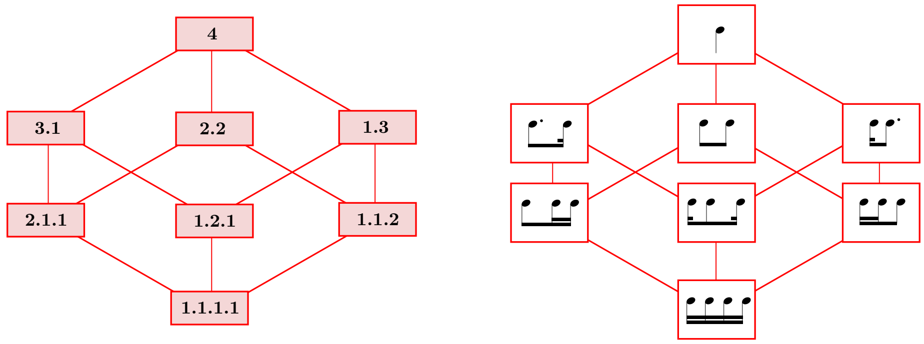


Figure 12: *Two isomorphic Hasse diagrams: the compositions of integer 4 and the eight possible rhythmic integer subdivisions of a quarter note.*

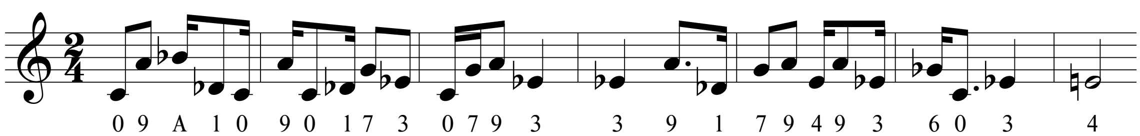


Figure 13: *A melodic fragment generated by the semi-open compositional system for* $j = 12$*,* $n = 100$*, and* $p = 0.5$*. The numbers below each note indicate the corresponded pitch class in integer notation.*

Each value of the list will be assigned to the *note.Note.quarterLength* object.

The length of the list *ritmos* will be passed as a parameter (*size*) to the binomial module of the system, which is responsible for the pitch configurations. The *size* corresponds to the number of tests. Two other parameters are inserted into the binomial (or pitch) module by the composer: the number of trials of each experiment (n) and the probability of each trial (p). The result is filtered through an octatonic converter and through a mod 12 operator. These modules perform respectively the conversion and the octave offset operations previously defined at the beginning of this section[47]. The result will also be inserted into the *note.Note* object of the *music21* package.

In order to generate a melodic fragment, I have chosen $j = 12$, for the uniform distribution module (assigned to rhythm), and $n = 100$ and $p = 0.5$, for the binomial distribution module (assigned to pitch). This is analogous to a probabilistic experiment consisting of flipping a coin 100 times. If this experiment is performed *size* times, one may ask how many times the result will be 1 head, 2 heads, 3 heads, and so on. As I have mentioned, the value of *size* in this system comes from the rhythmic module. The result is show in Figure 13. A histogram of the output reveals a shape typical of a binomial distribution. From this histogram one can easily recognize a centricity in A.

As it was done with the fragment for the open system, I also propose to harmonize this fragment. This time, for clarinet trio (two *B*♭ clarinets and a *B*♭ bass clarinet). The harmony will be extracted from the melodic line using the criteria of complementary harmony taking as a reference the same octatonic scale of the melodic line. The window size will be one measure, which means that every measure will have an octatonic aggregate, i.e., the entire octatonic scale used in the melody. The result is show in Figure 15.[48]

The third type of compositional system, according to our taxonomy, is the feedback system.

[47] Those are respectively operations 12 and 10 in the list of operations given at the beginning of this subsection

[48] A midi rendition is available at https://gitlab.com/musmat/open-compositional-system/-/blob/master/binomiamelodia2_.wav

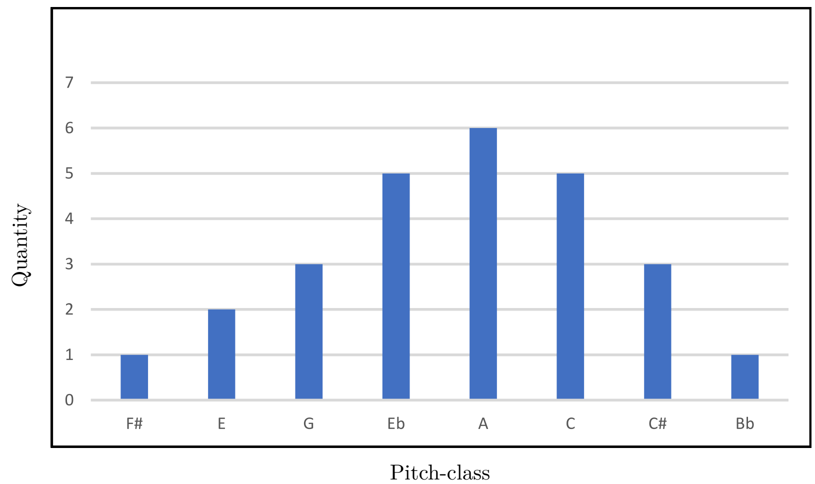


Figure 14: *The histogram of the system output revealing the binomial archetype and indicating a centricity in A.*

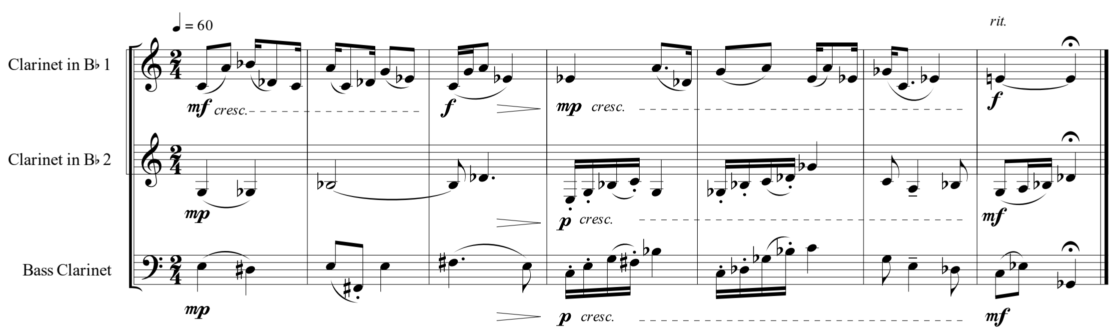


Figure 15: *The fragment generated by the semi-open compositional system harmonized through the criteria of complementarity (with the octatonic scale of the melodic line).*

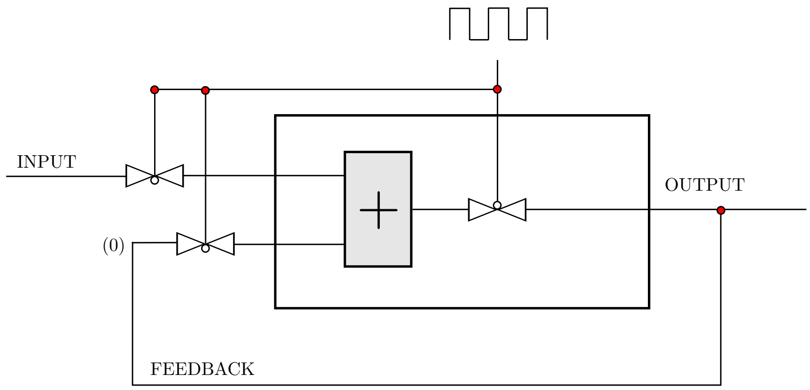


Figure 16: *A feedback compositional system with an addition operator*

This type of system has the output reinserted in the input, establishing what is known as *iterative process*.[49] There are two possible subtypes of input: a stream of information (a MIDI file, for example) or an initial trigger (an initial value employed only to start the process). The compositional system shown in Figure 16 has as its input a MIDI file (the melodic line of the previous example will be used). The output of the system is reinserted into its input. Both the MIDI file and the feedback information are controlled by transmission gates.[50] A square wave controls the cycle input/output of the system, in such a way that when its value is 1, the input and feedback ports are allowed to flow into the system and the output is blocked; when its value is 0 the input is blocked and the output flows. The system has one only operator that adds both the MIDI file and the feedback and apply mod 12. Figure 17 shows a step-by-step cycle of operation for the first five pitch classes inserted into the system. At the start, the MIDI file has pitch class 0 and, as the feedback has no value yet, an initial temporary value is given (0). These two values enter the system and are added, yielding 0. This value flows to the output and is sent back to the input. At this point the MIDI file has the pitch class 9, which is added with 0 (feedback value) yielding 9, which is sent to the input again and added with the next pitch class read from the file (10), yielding 7 (19 mod 12), and so on. Figure 18 shows the fragment produced by this compositional system. The contours of both melodic lines (input and output) are shown in Figure 19. In those graphs, the x-axis corresponds to each event (i.e., first pitch class, second pitch class, and so on) and the y-axis corresponds to the pitch class value.

The other subtype of feedback compositional system falls into a category known as chaotic maps. Those maps may be classified in terms of the number of their space dimension. This is an important factor for musical applications because the system's output may be mapped onto musical parameters. The output of a bidimensional chaotic map can be assigned to pitch and rhythmic parameters, for example. With a four-dimensional map one may have pitch, rhythm, dynamics and preset timbres controlled by its output. There are several known chaotic maps: one-dimension (Gauss, Logistic, Lambic, etc.), two-dimension (Hénon, Mandelbrot, Lozi, etc.), three-dimension (Lorenz, Ueda, Shimizu-Morioka, etc.) and four-dimension (Hyper-Lorenz, Hyper-Rössler).[51]

[49] According to Miranda (2004, p.83), "an iterative process is defined as a rule that describes the action that is to be repeatedly applied to an initial value x_0. The outcome of an iterative process constitutes a set, technically referred to as the orbit of the process"[27].

[50] A transmission gate works as an AND logic gate through which regular information (besides binary information) may flow.

[51] It is also possible to find five-dimension implementations[40]

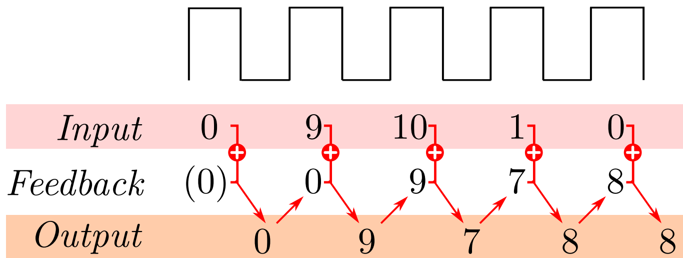


Figure 17: *Tracing the pitch class data within a feedback compositional system.*

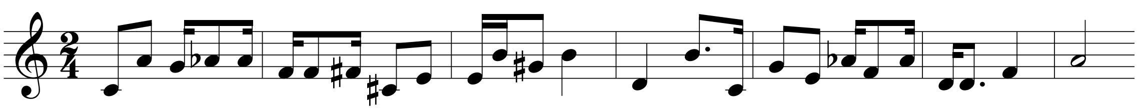


Figure 18: *The output fragment of our feedback compositional system using as input the melodic fragment of the previous compositional system.*

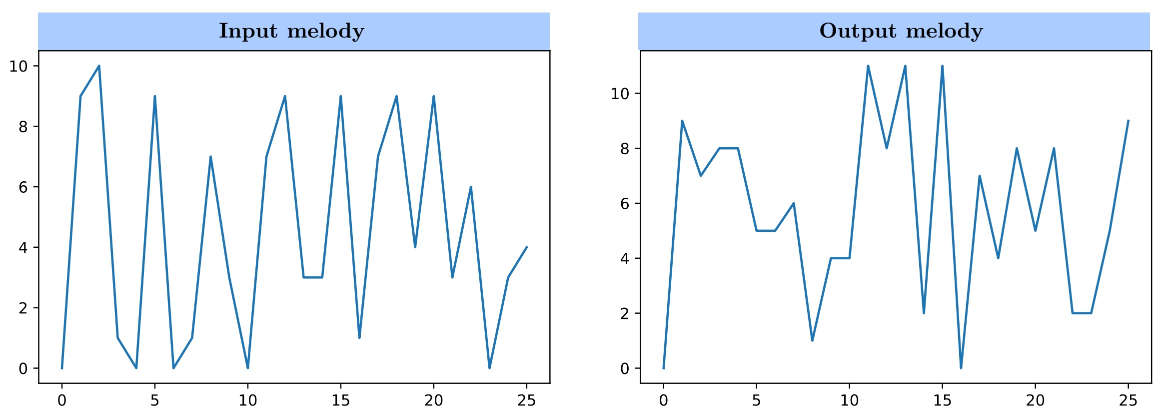


Figure 19: *The contours of the original melodic line inserted into the input of the feedback compositional system (left) and the resulted melody (right)*

Table 1: *The values of $f(z)$ for $c = 1$ and $c = -1$.*

$c = 1$		$c = -1$	
z_n	z_{n+1}	z_n	z_{n+1}
0	1	0	-1
1	2	-1	0
2	5	0	-1
5	26	-1	0
26	677	0	-1
677	458330	-1	0

I will select the Mandelbrot map, which has already been largely applied to the musical domain ([5], [39], [15], [18]). The Mandelbrot map is a representation in the complex plane of a Mandelbrot set M, "which is defined as the set of c $\in \mathbb{C}$ for which the sequence $c, c^2 + c, (c^2 + c)^2 + c, ...$ does not tend to ∞ as n tends to ∞" [6, p.75]. Graphically, this set is located inside the black region of the fractal structure shown in Figure 20, which is built computationally through the iteration of the function $z_{n+1} = z_n^2 + c$. For $z_0 = 0$, in each iteration, if $f(z)$ does not tend to infinity c is a member of the Mandelbrot set.[52] Table 1 shows the values of z_{n+1} for $c = 1 + 0j$ and $c = -1 + 0j$. In the first case the function tends quickly to infinity and, therefore, $1 + 0j$ does not belong to the set; in the second case the function is bounded and, so, $-1 + 0j$ belongs to the Mandelbrot set. Algorithm 1, shown below, receives the real and imaginary components of a complex number, tests if this number makes the function "to explode" under a certain number of iterations (which is also a value sent to this function), and returns the number of iterations, the complex number and its modulus. The number of iterations will be used to fill in an array which will correspond to the color of pixels in a screen. The black pixels correspond to the complex numbers that keep the modulus of z equal or smaller than 2 under iteration. The real and imaginary components of the members of the Mandelbrot set are extracted and assigned to pitch and duration (after a normalization). The result (after metrical adjustments) is shown in Figure 21. One can clearly see the intrinsic symmetry of the melodic line, a characteristic already presented in the fractal (Figure 20).

Algorithm 1 Mandelbrot's algorithm

procedure MANDELBROT (R, I, max_iter)
    $c \leftarrow complex(R, I)$
    $z \leftarrow 0.0j$
    $counter \leftarrow 0$
    while $abs(z) <= 2$ and $counter <= max\_iter$ **do**
        $z \leftarrow z^2 + c$
        $counter \leftarrow counter + 1$
    return $count, c, abs(z)$

[52] It is important to mention that if the modulus of z ever becomes larger than 2, the result will escape to infinity. Therefore, the set is formed by the complex numbers that remain inside a region centered at the origin with radius 2 [6, p.81].

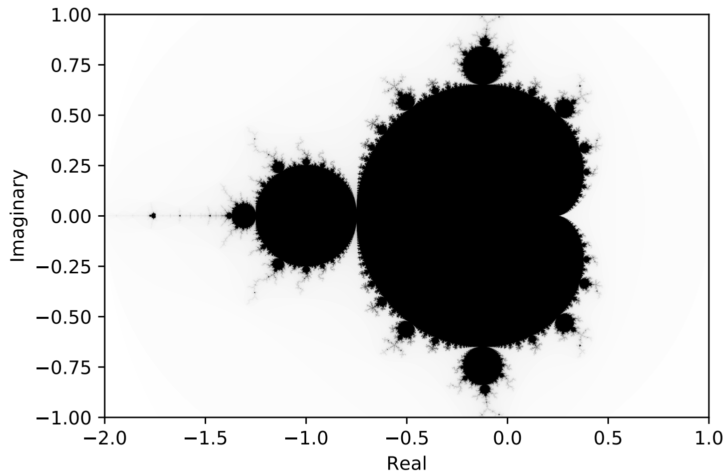


Figure 20: *The Mandelbrot Fractal*

Figure 21: *A possible musical representation of the Mandelbrot Fractal*

IV. Concluding Remarks

I have walked through various topics related to systems science and compositional systems. As it happens to all theories developed by humans, the theory of compositional systems, which appeared around 2009, is a work-in-progress and constantly benefits from the academic and artistic interchange with researchers and composers, as well as from the contributions given by my undergraduate and graduate students, who constantly collaborate with new ideas and pose new questions and challenges that require adjustments and yield expansions. Therefore, I believe that a mature formalization of this theory will be proposed in the near future. At this moment, our role is to support it with experiments, research, and reflections in order to gradually bring it to a more comprehensive, embracing and flexible state. One must remember that arithmetic and probability were formalized as late as 1889 and 1931, respectively by Peano and Kolmogorov.

At this point, the focus is primarily the steady-state compositional systems in open, semi-open and feedback formats. Exceptions are the few experiments with permutation systems (mobile forms) conducted by one my graduate students.[53] However, in those permutation systems, the score, once produced, is fixed, and the various possibilities appear only during a performance. A future goal includes research toward an elaboration of a score that could present changes over time.

Finally, I am very grateful to my friends in mathematics and music research who solved many of my questions related to formalization and notation during the process of writing this paper: Petrucio Viana (UFF), Carlos Almada (UFRJ) and Francisco Aragão (UFC), who read the text and sent detailed suggestions, and mainly Hugo Carvalho (UFRJ) who, in addition to reading the entire text, carefully studied several mathematical aspects and discussed the best strategies to make the formalizations clear and precise.

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